OUR INDUSTRY TODAY

Production Evaluation Techniques Based on Lactation Curves[1]

C. P. VAN TASSELL,[2,3] L. R. JONES,[3,4] and S. W. EICKER[5]
Cornell University
Ithaca, NY 14853

ABSTRACT

Test day data on milk yield for individual cows were generated using Monte Carlo simulation consisting of 1000 herds. Each herd contained 30 second lactation cows for each of 2 yr. Three scenarios were simulated with increasing effects of test day and seasonality. For each test day, several statistics were calculated for each cow: test day data on yield deviated from expectations, deviated test day data on yield transformed to account for correlation of consecutive test day data on yields, 305-d mature equivalent estimates, and changes in these values from the previous test day. A probability value for each herd was calculated for test of month of lactation effects using ANOVA models with and without cows in the model. No month of lactation effects were simulated. The distribution of generated probability values were tested for uniformity using a chi-square test. The distribution of probability values associated with the change in test day deviations were most nearly uniform, and results for these variables were similar when the cow effect was removed from the analysis model. The transformed variables also provided a fairly uniform set of probability values, although interpretation of these statistical tests was more difficult. Tests based on mature equivalent, 305-d records were oversensitive.
(**Key words**: lactation curves, mature equivalent, 305-d values)

Abbreviation key: **CDR** = Cornell Dairy Records, **ME** = 305-d mature equivalent, **MOL** = month of lactation, **P-P** = probability-probability, **TDD** = test day deviation of yield, **TDDA** = TDD based on actual DIM, **TDDM** = TDD based on midpoint DIM of MOL, **TDY** = test day data on yield.

INTRODUCTION

Accurate evaluation of milk yield in a dairy herd is complicated. Milk yield of individual cows is influenced by many factors, including genetic ability, parity, DIM, age, physiological state, and management (e.g., nutritional status or environmental conditions). Milk yield of a dairy herd is a composite value for the cows in that herd. Consequently, herd yield is influenced by individual cow effects in addition to seasonality of yield, age structure of the herd, and quality of overall management.

Two main factors should be considered when herd yield is evaluated. First, short-term (e.g., change in ration) or long-term changes (e.g., a change in heifer raising program) can be evaluated. Second, the magnitude of yield or the shape of the lactation curve can be considered.

This study examines methods for evaluation of long-term changes in the shape of lactation curves. An example of such an analysis might be to compare second lactation freshenings in a herd with a regional standard or a previous evaluation of the same herd to examine differences that are due to a change in management (e.g., grouping first lactation cows separately, changing rations for a specific group of cows, or changing management of dry cows).

One technique for evaluation of long-term changes in the shape of the curve for milk yield is to analyze composite lactation curves.

Received May 13, 1994.
Accepted November 10, 1994.

[1]This research is part of the NC-119 Regional Project, Dairy Herd Management Strategies for Improved Decision Making and Profitability.

[2]Current address: A218 Animal Sciences, University of Nebraska, Lincoln 68583-0908.

[3]Department of Animal Science, College of Agriculture and Life Sciences.

[4]Reprint requests.

[5]Department of Clinical Sciences, College of Veterinary Medicine.

Lactation curves are usually developed using the average test day data on milk yield (**TDY**) grouped in monthly intervals of DIM within parity groups (1, 2, and ≥3). Lactation curves have been used to estimate peak yield and postpeak persistency. Persistency estimates may be biased by culling of poor cows and by including partial records. If cows that yield above or below average tend to have incomplete lactations, because of random sampling or some systematic cause, the early monthly averages will be overestimated or underestimated relative to the complete lactations. Because of this potential problem, Galligan et al. (7) criticized use of lactation curves to estimate persistency. In addition, because each point estimated in the composite lactation curve includes test day observations collected over 30 d, the precision of the estimate of peak yield is limited. Finally, because each month of lactation (**MOL**) may represent different cows, estimates of persistency may be confounded by the group of cows that have complete or incomplete lactations.

Standardized mature equivalent, 305-d (**ME**) yield for each cow can be used to evaluate short-term changes in productivity. Factors based on age, herd yield, and season of calving have been developed to extend lactations to 305 d (4, 15). Underlying these projection factors are implicit lactation curves. Additional correction factors based on age have been developed to adjust the 305-d yield to a mature equivalent basis (8, 14). These ME records allow comparison of cows of different ages, which is especially useful for genetic evaluations and decisions on culling. Two primary criticisms of this approach exist. First, the same set of factors is used for all herds of similar yield, regardless of postpeak persistency. Herds with poor persistency have early lactation ME values biased in the positive direction. Second, extension factors currently in use were estimated in the 1970s when average yield was lower than for most herds in the 1990s [e.g., the high herd average group in adjustment factors used in the Cornell Dairy Records (**CDR**) is for herd averages of ≥7045 kg (14)]. Because CDR ME factors were estimated by yield groupings, these factors may not be appropriate for herd levels not represented in the data used to calculate those estimates. Factors not reflecting changes in productivity could lead to biased ME values. Consequently, ME values in early lactation may be underestimated for high yielding cows.

Changes in milk yield have been evaluated using the incremental change in ME (7). This technique assumes that cows without management changes maintain a nearly constant ME yield during a lactation. Thus, monthly comparisons of the changes in ME yield indicate changes in management. Although this technique can be biased by problems with ME adjustments, the bias is reduced by evaluation of differences in sequential values.

The effect of test day management estimated using a test day model described by Everett and Schmitz (6) may be the best tool to analyze management changes over time. In addition, changes in individual cow "residuals" should also be a valuable measure for monitoring specific cow performance. However, this information is only routinely available for herds processed at the CDR. In addition, that information is available only through a proprietary personal computer software system. As a result, the information on test day management is available only to a small fraction of producers or consultants. Therefore, management information based on traditional yield information is the only option for most dairy producers.

The objective of this study was to evaluate systematically the accuracy (i.e., specificity) of seven different evaluation techniques based on lactation curves using a Monte Carlo simulation approach. Simulation was designed to model three herd scenarios with varying degrees of test day (herd) effects and seasonal calving patterns. In every simulation, the underlying DIM effects were identical, and no deviations from the DIM effects were simulated. Each variable corresponded to a technique of lactation curve analysis and was used to test for a deviation from a standard lactation curve in a proxy variable for DIM, MOL, a 30.5-d interval of DIM, starting at d 7. Therefore, testing for deviations from a standard lactation because of MOL also tests for deviations caused by DIM. Because no DIM deviations were generated, the primary concern with each variable was the probability of making a Type I error, i.e., concluding that a significant effect is present when none exists. This probability was equivalent to rate of false-positive results for each technique.

MATERIALS AND METHODS

Monte Carlo Simulation

Each simulation consisted of 1000 herds, each containing 30 second lactation cows freshening annually for each of 2 yr. Cows were normally considered dry at 60 d before the subsequent freshening date. Days open were generated as 50 + G, where G is distributed as a gamma (α = 5.0; β = 9.0) random variable (3), using the algorithm described by Ahrens and Dieter (2) as implemented by B. W. Brown and J. Lovato (unpublished data, 1992, StatLib file/general/ranlibf.uuen). The main (uniform) random number generator used in this set of subroutines is based on the algorithm of L'Ecuyer and Côté (10). Mean and standard deviation of days open were 95 and 15 d, respectively; the minimum was 50 d. Data included only records for cows that were currently in the second lactation and cows that had completed their second lactation but had not freshened for the third time. These data correspond to those available electronically to a consultant using the remote management system at CDR.

Three models were used to simulate the data. Each phase increased in complexity.

Phase 1. The first simulation model was

$$y_{ij} = DIM_i + cow_j + error_{ij}, \qquad [1]$$

where

y_{ij} = TDY at i DIM for cow j at testing period k,
DIM_i = fixed effect representing average milk yield at i DIM,
cow_j = random cow effect representing yield differences of cows, and
$error_{ij}$ = random residual.

Fixed effects for milk yield at i DIM were based on second lactations for cows in herds with rolling herd averages from 8844 to 9977 kg (13). Cow effects were distributed normally (1) with mean 0 and standard deviation 2 kg. Finally, the residual effects were sampled from a multivariate normal distribution (0 ± 6 kg), and corr($error_{ijk}$, $error_{ijk'}$) = $\rho^{|k-k'|}$ if k ≠ k', where ρ is the autocorrelation parameter. The value of ρ was .73, based on work by R. W. Everett et al. (1993, unpublished data). Calving dates were distributed uniformly throughout each year for the first phase of the simulation. If a milk weight was negative, the cow was considered to be dry on that date.

Phase 2. The simulation model used in the second phase was

$$y_{ijk} = DIM_i + cow_j + TD_k + error_{ijk}, \qquad [2]$$

where

y_{ijk} = TDY at i DIM, for cow j, on testing day k,
DIM_i = fixed effect representing average milk yield at i DIM,
cow_j = random cow effect representing yield differences of cows,
TD_k = random effect of test day (herd) common to all cows in the herd milking on test day for period k, and
$error_{ijk}$ = a random residual.

Effects common to Model [1] were simulated in the same manner. Herd effects were normally distributed and mutually independent (0 ± 1.5 kg). Calving dates were distributed uniformly throughout each year. As in phase 1, if a milk weight was negative, the cow was considered to be dry on that date.

Phase 3. The simulation model used in the last phase was

$$y_{ijkl} = DIM_i + cow_j + TD_k + season_l + error_{ijkl}, \qquad [3]$$

where

y_{ijkl} = TDY at i DIM for cow j on test day k for testing date l,
DIM_i = fixed effect representing average milk yield at i DIM,
cow_j = random cow effect representing yield differences of cows,

TD_k = random effect of test day (herd) common to all cows in the herd milking on test day period k,
$season_l$ = fixed season of test day effect for a TDY recorded on date l, and
$error_{ijkl}$ = random residual.

Effects common to Models [1] and [2] were generated in the same way. The seasonality effect, $season_l$, was constant for all years at +1 kg for milk yields recorded from September to January and −1 kg for milk yields from February to October. Finally, seasonal calving was added; 75% of the calvings were in September to November in each year. Calving dates were distributed uniformly within the calving periods, i.e., from September to November or December to October. As in phases 1 and 2, if a milk weight was negative, the cow was considered to be dry on that date.

Data Analysis

Record Adjustments. Estimates of lactation to date were calculated for each TDY recorded for a cow, up to 305 DIM. Values of lactation to date were used to project a 305-d yield. Finally, age factors were used to calculate an estimate of ME-adjusted lactation for every cow on each test day. All procedures were based on those used by CDR (4, 14).

Data Analyzed. Each TDY was assigned to MOL based on DIM on the day that the observation was recorded. Expected milk yields were calculated using the solutions from Stanton et al. (13). Each observed TDY was deviated from the predicted values to obtain test day deviation of yield (**TDD**). The TDY were deviated from average yields using expected yields based on actual DIM (**TDDA**) or DIM based on the midpoint of the appropriate MOL (**TDDM**).

Because data with correlated residuals result in an inappropriate statistical test (11), an attempt was made to account for the correlation of the residuals. The adjustment method is described in Appendix 1.

In addition to the observed and adjusted variables used, the changes in these variables on consecutive test days and the changes in observed TDY were also examined for test days ≥2.

The large number of variables was included to determine the effects of 1) using measures of lactation curve with values based on ME, 2) using approximated DIM to group yields, 3) using autocorrelation of residuals to determine whether the suggested adjustment is necessary and effective, and 4) using changes in variables to account for cow effects.

Statistical Analysis. The general linear models procedure of SAS (12) was used for statistical analysis. The first model used to analyze the data included cow, MOL, and their interaction. Sequential, SAS Type I sums of squares were used. The MOL was tested using the mean square for interaction of cow and MOL. The second model excluded cow and interaction; as a result, MOL was tested using the residual mean square. A probability value was calculated for each herd and model combination for every simulation phase.

Because no effects of MOL were simulated in any of the data, the null hypothesis was always true. Therefore, given the definition of a probability value (i.e., the probability that a more significant test statistic would occur by chance under the null hypothesis), the probability values should be distributed as a continuous standard uniform distribution (i.e., all probability values from 0 to 1 are equally likely under the null hypothesis). The distribution of probability values for each response variable and analysis model was compared with a uniform distribution in several ways. First, a probability-probability (**P-P**) plot (9) was used as a subjective measure. The P-P plot, in this case, is simply the sorted probability values plotted against their expected values. The expected value for sorted value i is (i − .5)/n, where n is the number of probability values. Second, a chi-square test for goodness of fit was used to assign an objective value on how closely the observed probability values followed the uniform distribution. The chi-square test was calculated using the formula

$$\chi^2_c = \sum_{i=1}^{50} \frac{(obs_i - exp_i)^2}{(exp_i)}$$

where obs_i is the number of probability values in range i of values (0 − .02, .02 − .04, . . . , .98 − 1), and exp_i is 1000/50 = 20.

RESULTS AND DISCUSSION

TDD Variables

Change Versus Observed Variables. The change in a measurement of milk yield on consecutive test days generally resulted in a statistical test with better characteristics than a test of the original variable, based on subjective observation of P-P plot (e.g., Figure 1) and objective measurement using test statistics for chi-square goodness of fit (Table 1). The tests for uniformity of probability values based on change in TDDA were the only nonsignificant tests (α = .01) for all models and all phases (Table 1). The improvement observed when the change variables were used suggests that these variables have better statistical properties than the values for an individual test day and, if possible, should be used. Little improvement occurred when deviations were used for TDDA and TDDM variables adjusted for the autoregression.

The mean, median, and decile counts for probability values, rather than P-P plots for each variable and model, are included in Table 1. These values provided most of the information contained in the P-P plots.

Adjustment to Account for Autoregression of Residuals. For nearly all response variables, the probability values based on tests of the independent transformed variables were more uniform (Table 1). The probability values using independent test day observations and change in independent test day observations were very nearly uniform for all phases when cows were included in the analysis model. The only variable that did not have more uniform probability values after transformation was TDDM without cows in the analysis model.

The tests based on adjusted TDDM were more insensitive than the TDDM. The differences in sensitivity can be seen in Table 1 through not only the mean and median probability values for analysis models including and excluding cow, but also through the large number of probability values in deciles 6 to 10 for both analysis models. The independence adjustments worked well for the data in which cow was included in the ANOVA model; however, when cow was removed from the model, the tests were too conservative.

Because the data analyzed in the ANOVA model represented a transformation of the test

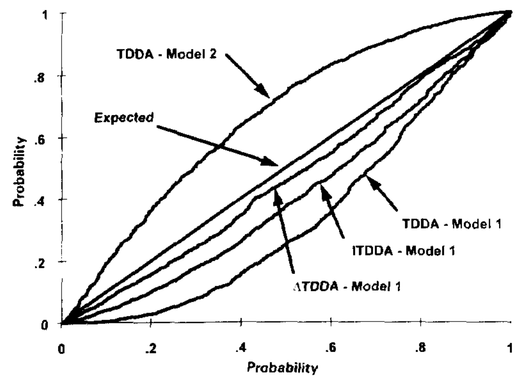

Figure 1. Example of a probability-probability plot for several combinations of response variables and models compared with the expected distribution of probability values (TDDA = test day deviation of yield based on actual DIM, ΔTDDA = change in TDDA, ITDDA = independent (adjusted for autoregression) TDDA, Model [1] includes cow, month of lactation, interaction, and residual, and Model [2] includes only month of lactation and residual).

day data, interpretation of the results for management decisions is difficult. In addition, because the transformed data did not perform substantially better than the change in TDD variables, use of transformed data may not be justified.

Midpoint Versus True DIM. Differences were found in the responses to the approximation of using expected yield corresponding to the DIM of the midpoints of the MOL rather than the actual DIM on which the TDY was measured. The TDD and change in TDD for statistical tests were more sensitive when midpoint DIM were used. The statistical tests based on independent TDD and change in independent TDD, however, were less sensitive when midpoint rather than true DIM was used to determine expected yield. For most variables, the use of midpoint (approximate) rather than actual DIM tended to make the probability values less uniform, suggesting that the actual DIM should be used, if possible.

Analysis Models. In general, changing from the model that included cow and interaction of MOL and cow to the one without, resulted in a less sensitive statistical test, probably because the variation accounted for by cows was added to the residual effect, increasing mean squared error, which often resulted in a smaller test statistic. The difference between the two types of models was smaller for the data based on changes in TDY, because much of the cow effect was removed as a record from a cow was subtracted from another record on the same cow, reducing the differences in overall cow productivity.

TABLE 1. Mean, median, decile counts, chi-square statistics, and probability values for the analysis models for each response variable (VAR) and phase of simulation.

VAR[1]	Model[2]	Mean	Median	Observations in decile 1	2	3	4	5	6–10	χ^2 Statistic	P
				Phase 1							
TDDA	1	.45	.43	182	112	102	77	85	442	285	<.01
	2	.72	.82	41	37	31	58	50	783	1507	<.01
TDDM	1	.40	.33	261	102	106	81	67	383	576	<.01
	2	.66	.75	67	50	35	69	60	719	805	<.01
ITDDA	1	.50	.49	106	98	98	105	104	489	46	.62
	2	.65	.71	25	42	61	81	83	708	339	<.01
ITDDM	1	.63	.73	106	64	44	49	68	669	600	<.01
	2	.83	.94	30	15	28	19	26	882	6170	<.01
ME	1	.00	.00	1000	0	0	0	0	0	49,000	<.01
	2	.05	.00	873	46	31	16	13	21	26,198	<.01
ΔTD	1	.03	.00	929	40	13	11	4	3	30,430	<.01
	2	.01	.00	971	22	5	0	2	0	38,472	<.01
ΔTDDA	1	.51	.52	99	109	90	93	96	513	31	.98
	2	.46	.45	128	116	115	105	88	448	60	.13
ΔTDDM	1	.38	.31	237	125	122	83	91	342	323	<.01
	2	.33	.26	293	140	109	94	85	279	703	<.01
ΔITDDA	1	.53	.55	109	93	76	98	70	554	62	.11
	2	.46	.44	164	96	108	94	92	446	114	<.01
ΔITDDM	1	.50	.51	174	87	66	77	83	513	291	<.01
	2	.48	.49	192	86	79	77	77	489	346	<.01
ΔME	1	.23	.10	493	135	81	58	60	173	3504	<.01
	2	.13	.03	669	101	68	52	28	82	9191	<.01
				Phase 2							
TDDA	1	.43	.40	225	100	90	85	73	427	496	<.01
	2	.70	.79	38	33	52	51	50	776	878	<.01
TDDM	1	.39	.34	281	109	77	100	61	372	789	<.01
	2	.65	.73	59	58	51	68	65	699	476	<.01
ITDDA	1	.47	.47	113	109	113	98	94	473	51	.39
	2	.63	.67	26	50	69	64	99	692	233	<.01
ITDDM	1	.60	.67	124	58	62	49	70	637	395	<.01
	2	.83	.94	25	14	11	24	29	897	5710	<.01
ME	1	.00	.00	1000	0	0	0	0	0	49,000	<.01
	2	.05	.00	856	49	37	22	16	20	24,488	<.01
ΔTD	1	.05	.00	872	59	26	16	9	18	23,521	<.01
	2	.02	.00	942	25	13	9	7	4	33,263	<.01
ΔTDDA	1	.49	.48	105	94	105	102	109	485	47	.55
	2	.47	.47	118	111	110	91	107	463	59	.16
ΔTDDM	1	.38	.34	209	146	104	111	74	356	306	<.01
	2	.35	.30	262	134	102	100	89	313	503	<.01
ΔITDDA	1	.52	.54	108	88	88	76	88	552	67	.04
	2	.47	.46	158	103	87	87	107	458	120	<.01
ΔITDDM	1	.50	.54	192	74	69	74	61	530	333	<.01
	2	.49	.52	198	80	68	57	80	517	422	<.01
ΔME	1	.25	.14	440	135	106	59	61	199	2803	<.01
	2	.16	.05	599	128	79	55	34	105	6400	<.01

(continued)

TABLE 1. (continued) Mean, median, decile counts, chi-square statistics, and probability values for the analysis models for each response variable (VAR) and phase of simulation.

VAR[1]	Model[2]	Mean	Median	Observations in decile 1	2	3	4	5	6–10	χ^2 Statistic	P
				Phase 3							
TDDA	1	.34	.25	332	115	110	69	65	309	1406	<.01
	2	.66	.74	58	47	58	59	73	705	609	<.01
TDDM	1	.26	.15	435	126	91	65	61	222	3297	<.01
	2	.62	.68	68	72	43	78	92	647	369	<.01
ITDDA	1	.41	.37	201	132	98	99	93	377	207	<.01
	2	.55	.56	86	87	92	90	90	555	98	<.01
ITDDM	1	.55	.60	142	72	63	62	81	580	250	<.01
	2	.51	.53	231	67	70	50	61	521	1187	<.01
ME	1	.00	.00	993	3	2	0	1	1	45,868	<.01
	2	.18	.06	574	121	67	63	49	126	6679	<.01
ΔTDA	1	.14	.04	632	125	84	50	33	76	7822	<.01
	2	.05	.01	879	51	29	20	11	10	21,537	<.01
ΔTDDA	1	.47	.46	128	122	114	67	119	450	68	.04
	2	.47	.45	129	117	103	98	98	455	52	.37
ΔTDDM	1	.40	.35	220	121	101	102	83	373	267	<.01
	2	.37	.30	240	134	126	86	78	336	344	<.01
ΔITDDA	1	.50	.51	133	107	75	92	81	512	97	<.01
	2	.43	.40	197	122	98	81	103	399	221	<.01
ΔITDDM	1	.50	.52	168	90	65	81	80	516	173	<.01
	2	.47	.47	185	110	79	87	57	482	297	<.01
ΔME	1	.36	.29	313	120	76	87	73	331	1044	<.01
	2	.20	.09	523	129	92	61	71	124	3977	<.01

[1]TDDA = Test day deviation of yield based on actual DIM, TDDM = test day deviation of yield based on midpoint DIM of month of lactation, ITDDA = independent (adjusted for autoregression) test day yield based on actual DIM, ITDDM = independent test day yield based on midpoint DIM of month of lactation, ME = mature equivalent 305-d yield, ΔTD = change in consecutive test day yield, ΔTDDA = change in consecutive TDDA, ΔTDDM = change in consecutive TDDM, ΔITDDA = change in consecutive ITDDA, ΔITDDM = change in consecutive ITDDM, ΔME = change in consecutive ME.

[2]Model [1] includes cow, month of lactation, interaction, and residual. Model 2 includes only month of lactation, and residual.

ME Variables

The analysis of the observed CDR ME variable yielded consistently oversensitive results. In all three phases, >90% of the replicates had probability values <.01. Several possible reasons exist for the large number of significant tests. First, average productivity of the cows changed (CDR adjustment factors were last calculated in the 1970s); this change could bias extended lactations, even if only height, and not shape, of the lactation curve changed, because multiplicative extension factors are used. Second, extension factors used in the CDR ME calculations were calculated with data including incomplete lactations (>180 DIM) (15), which may have caused differences that were due to culling to be included in data used to estimate adjustment factors. No culling was simulated in these data, and, as a result, the extension factors may have overestimated complete lactations when culling did not occur. Use of the change in CDR ME resulted in a test that was less sensitive but had a large proportion of small probability values (Table 1). Although many of the tests based on TDY performed more poorly as the simulation assumptions became more complex (realistic), the CDR ME and, especially, change in CDR ME variables performed better as the data simulation assumptions were more realistic, probably because these data were likely more similar to those used in calculation of these factors.

One alternative to use of the change in ME values is to adjust for the bias caused by the unrealistic ME adjustment factors (J. D. Ferguson, 1993. personal communication). One potential disadvantage of this method is that 305-d ME records do not have homogeneous variance (R. W. Everett, 1993, unpublished data); therefore, the assumptions needed for an analysis of variance to be valid may not be met. S. W. Eicker 1994 (personal communication) suggested that this might at times be a strength, rather than a weakness, because detection of problems early in lactation was more critical to profitable management systems. An alternative suggestion, which uses the ME system but should correct for the heterogeneous variance, is to calculate an expected milk yield by determination of the TDY that results in the ME yield corrected for bias (5). Although this procedure is valid, comparison of expected milk yield based on herds with similar productivity using estimated lactation curves directly is more logical than indirectly via the ME calculations. These methods should be equivalent if the extension and adjustment factors used in ME calculations were reestimated using contemporary data. Use of lactation curves, however, required the assumption that the herd did not change over time or only recent data were used and that data were sufficient for accurate estimates of the lactation curves.

CONCLUSIONS

Statistical tests for MOL for data based on TDD were reasonably accurate, although tests based on differences of consecutive test days performed better. The adjustment for autoregression worked well, although interpretation of these results was less clear. Use of expected milk yield based on approximate (midpoint of MOL) rather than actual DIM increased oversensitivity. Removal of cows from the ANOVA model made tests undersensitive for TDD and made little difference for changes in TDD. Tests based on ME records were not accurate. More results were significant than expected, probably because of old adjustment factors and possibly because of the simplicity of the simulation model. Although none of the tests proved to be ideal, change in TDDA seemed to perform best in this simulation project, because change in TDDA was the only variable for which probability values were not significantly different ($P > .01$) from the distribution expected. When these deviated values were used, cows did not need to be included in the ANOVA model.

If the shape of the lactation curve is of primary interest, changes in TDDA could be used to generate lactation curves by the choice of an (arbitrary) anchor point (e.g., average peak yield for all cows in a parity), and change in TDDA used to deviate from that point. This method would eliminate the potential biases in lactation curves that were due to culling and sampling but would base the curve on a statistically valid variable.

Although our results were based on an evaluation of lactation differences by herd, the validity of analogous variables logically should be similar in evaluation of individual cow performance. Because of difficulty in interpretation, the variable transformed to account for independence would likely be of little use for evaluation of individual cows. Change in consecutive CDR ME values would appear to be of uncertain value based on these data because a large fraction of herds had statistically significant differences in CDR ME yields, but no such differences were simulated. Apparently, differences of consecutive TDD (based on actual DIM) would be a likely choice for an evaluation of individual cows.

Finally, many different reasons exist for analysis of milk yield and those reasons for analyzing yield may suggest different tools. For example, if evaluation of short-term changes in lactation were of interest, changes in TDD or ME might be appropriate. Alternatively, if the shape of the curve for milk yield over time is of interest, then lactation curves would be appropriate.

ACKNOWLEDGMENTS

The random number subroutines were from StatLib, an electronic archive of statistical algorithms and datasets distributed by electronic mail, Gopher, or FTP (file transfer protocol). For retrieval instructions and an index of available software and data, send the one line e-mail message 'send index' to statlib@lib.stat.cmu.edu.

REFERENCES

1 Ahrens, J. H., and U. Dieter. 1973. Extensions of Forsythe's method for random sampling from the normal distribution. Math. Comput. 27(124):927.

2 Ahrens, J. H., and U. Dieter. 1982. Generating gamma variates by a modified rejection technique. Commun. Assoc. Comp. Machinery 25:47.
3 Casella, G., and R. L. Berger. 1990. Statistical Inference. Wadsworth and Brooks/Cole, Pacific Grove, CA.
4 Dairy Records Processing Laboratory. 1975. Supervisors Handbook, Chapter 7 for Misc. Topics. Cornell Univ., Dairy Records Process. Lab., Ithaca, NY.
5 Eicker, S. W., S. Stewart, C. L. Guard, and Y. T. Gröhn. 1993. Prediction of the next sample day production—the creation and validation of expected milk. J. Dairy Sci. 76(Suppl. 1):268.(Abstr.)
6 Everett, R. W., and F. Schmitz. 1994. Dairy Genetics in 1994 and Beyond: Cow and Sire Evaluation Using Test-Day Records, DairyGene, and DairyView for Dairy Farm Management. Anim. Sci. Mimeo Ser. 170. Dep. Anim. Sci., Cornell Univ., Ithaca, NY.
7 Galligan, D. T., J. D. Ferguson, W. Chalupa, and C. F. Ranmberg. 1992. Quantifying changes in milk production: don't let someone throw you a curve. Compend. North Am. Ed. 14:549.
8 Keown, J. F., and R. W. Everett. 1984. Age-month adjustment factors for milk, fat, and protein yields in Holstein Cattle. J. Dairy Sci. 68:2664.
9 Law, A. M., and W. D. Kelton. 1991. Simulation Modeling and Analysis. McGraw-Hill, New York, NY.
10 L'Ecuyer, P., and S. Côté. 1991. Implementing a random number package with splitting facilities. Assoc. Comp. Machinery Trans. Math. Software 17:98.
11 Neter, J., W. Wasserman, and M. H. Kutner. 1985. Applied Linear Statistical Models. 2nd ed. Irwin, Homewood, IL.
12 SAS® User's Guide: Statistics, Version 5 Edition. 1985. SAS Inst., Inc., Cary, NC.
13 Stanton, T. L., L. R. Jones, R. W. Everett, and S. D. Kachman. 1992. Estimating milk, fat, and protein lactation curves with a test day model. J. Dairy Sci. 75:1691.
14 Wadell, L. H. 1979. Holstein Projection and Age-season Adjustment Calculations at the Northeast DRPL. Dairy Records Processing Laboratory, Cornell Univ., Ithaca, NY.
15 Wiggans, G. R., and L. D. Van Vleck. 1979. Extending partial lactation milk and fat records with a function of last-sample production. J. Dairy Sci. 62:316.

APPENDIX 1

An approximation was used to generate adjusted TDD with uncorrelated residuals. The method is an approximation because the mean of the observations is not zero and must be estimated. The steps involved in calculating the independent data for a cow are

1. $$c = \sqrt{\frac{1}{1-\rho^2}}$$

2. $$\overline{D} = \sum_{i=1}^{n} (TD_i - \overline{M}_i)$$

3. $$ITD_1 = TD_1 - \overline{M}_1$$

4. $$ITD_i = c((TD_i - \overline{M}_i - \overline{D}) - \rho\,(TD_{i-1} - \overline{M}_{i-1} - \overline{D}))\ i = 2, \ldots, n$$

where TD_i is TDY i for the cow, ITD_i is adjusted TDY i with independent residuals, $\overline{M}_i$ is average milk on the DIM of test day, and n is the number of test days observed for the cow. $\overline{M}_i$ can represent average yield on the actual test day or on the midpoint of the MOL.