GENETICS AND BREEDING

Productive Life Evaluations: Calculation, Accuracy, and Economic Value

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ABSTRACT

A preliminary national evaluation of productive life was computed from records of 11 million Holsteins. Data were a mixture of projected and completed months in milk by 7 yr of age. Data were analyzed with the animal model programs currently used for yield traits. Genetic trend for productive life was positive as a correlated response to past selection for yield and type traits. Phenotypic trend was slightly negative. Active AI bulls had PTA that ranged from −1.7 to +4.3 mo and mean reliability of 56%; 3-yr-old cows had mean reliability of 29%. Yield and type PTA were almost uncorrelated with productive life PTA for active AI bulls but had positive correlations (.25 to .36) for all bulls born since 1980. An index that combines yield and productive life PTA with relative weights 2.5:1 can increase economic progress by up to 4%. Nearly identical progress can result from an index that combines yield and productive life adjusted for yield. Multitrait evaluations might produce higher reliabilities for productive life by inclusion of correlated traits measured earlier in life such as yield, type, and somatic cell score.
(**Key words**: productive life, herd life, genetic evaluation, economic value)

Abbreviation key: **APL** = PL adjusted for milk yield, **PL** = productive life, $\mathbf{PTA_{APL}}$ = PTA for APL, $\mathbf{PTA_{M}}$ = PTA for milk yield, $\mathbf{PTA_{PL}}$ = PTA for PL, **REL** = reliability, **SCS** = somatic cell score, $\mathbf{v_{M:APL}}$ = relative value of milk yield to APL, $\mathbf{v_{M:PL}}$ = relative value of milk yield to PL.

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INTRODUCTION

Selection programs for dairy cattle often have not included traits that directly measure how long the cow will live. Herd life is an important economic trait; however, its heritability is low, and herd life is expressed at a later age than the traits in current selection programs.

VanRaden and Klaaskate (19) proposed direct selection for productive life (**PL**), a trait defined as total months in milk through 84 mo of age. A method to include predicted PL as a data source was developed so that genetic evaluations could be calculated earlier in an animal's life. Predicted PL data are analogous to predicted milk records and allow analyses to include early data without assumptions about distribution. Such linear model analyses might evaluate a mixture of censored and uncensored records nearly as well as nonlinear approaches (7, 8), but with reduced computation (12), faster variance estimation, simpler hypothesis testing, and easier explanation.

Previous evaluations of herd life often were based on binomial data. Survival to 48 mo of age (9) or to each additional calving year (12) was included in BLUP sire models. Increases in economic gains of 2 to 4% from selection were expected for indexes that incorporated measures of stayability (14).

The trait PL has a much more continuous, although not quite normal, distribution (19). For younger cows, PL records tend to be bimodal (19) because records from cows that are already dead include only past months in milk, but records from cows that are still alive also include expected future months in milk. VanRaden and Klaaskate (19) suggested methods to evaluate PL from completed or predicted data but did not report on the reliability (**REL**) and availability of PL evaluations for current animals or on the use of such evaluations in an index.

Herd life has higher economic value than many traits currently evaluated and may be about one-third as important as yield, based on an average of recent estimates (1, 3, 4, 10, 15, 18). Reasonable genetic progress for herd life can be achieved by selection for yield and type traits that are correlated with herd life (2, 6). However, genetic gains may not lead to phenotypic gains for herd life (5). Future cows may be healthier, higher producing, and more functional, but producers likely will raise their culling standards rather than increase herd life of cows.

Genetic evaluations for PL measure differences among months of milk for cows in the same herd at the same time and reflect culling of all types. To isolate culling based on factors other than yield, several researchers (1, 2, 8, 16, 17) have suggested the removal of the correlated effect of yield when herd life is evaluated. An economic index that combines yield, herd life, type traits, and somatic cell score (**SCS**) is needed to avoid underemphasis, overemphasis, or use of independent culling for individual traits by breeders. Single-trait prediction of these PTA may make their combination into one index more difficult and less accurate than for multiple-trait prediction.

The objectives of this study were to evaluate national PL data with an animal model, to describe properties of the evaluations, to compare PL and PL adjusted for milk yield (**APL**), and to develop an index to combine evaluations of PL with other traits.

MATERIALS AND METHODS

Animal Model

Data for the PL evaluation were constructed from lactation records available during November 1992 for calculation of yield evaluations released by USDA in January 1993. Only results from the joint analysis of the Holstein and Red and White breeds are reported. The 11.1 million cows with first lactation records and born from January 1960 through November 1989 were included.

Complete PL was measured as a cow's total months in the milking herd for the first 7 yr of life with a limit of 10 mo per lactation. For cows that changed herds, PL was summed across herds, and management groups were assigned based on the first herd. Grouping across time would be more precise with the models of Ducrocq (7), Ducrocq et al. (8), and Madgwick and Goddard (12), because groupmates are assigned on the basis of herd-year or herd-year-season rather than for life.

Predicted PL was computed for any cow still alive or that had been sold for dairy purposes at <7 yr old. Predictions were by multiple linear regression on cumulative months in milk, months of milk during the current lactation, current months dry (including months in milk >10), current milking status (fresh or dry), and age at first calving (19).

Records of cows that had an opportunity to reach 7 yr of age received a weighting of 1. For completed and projected records of cows born during the last 7 yr (since November 1985), deviations from management group were expanded to stabilize genetic variance and given less weight in the animal model (20, 21). The projected records of cows with a last record coded as sold for dairy also were expanded and given less weight based on their age at date of sale. Expansion factors and weights were obtained by linear interpolation between those for 36, 42, 48, 54, 60, 72, and 84 mo computed by VanRaden and Klaaskate (19).

An animal model evaluation of PL was computed with the same model and programs as for yield evaluations from January 1993 (20). Management groups were formed for cows with first calvings in the same herd and with similar birth months and registration status. Random effects for genetic merit of the animal and interaction of herd and sire were included in addition to fixed genetic group effects for unknown parents. Because each cow expresses PL only once, permanent environmental effects were not needed. Permanent environmental effects remained in the computer programs, but solutions were effectively set equal to 0: their variance was assigned to be only .1% of phenotypic variance.

Heritability was assumed to be 8.5% (19), and interaction of herd and sire was assigned to be 5% of phenotypic variance. These fractions of variance are about one-third as large as the values assigned for yield traits (20). The estimate of heritability for PL was from a multiple-trait study (19) that accounted for yield selection, but the PL evaluations use only PL data in a single-trait system. Biased parameter estimates can result for either case if the true model actually is nonlinear or more complex (4).

Adjustments for heterogeneous variance were not applied, but standard deviations were estimated for region-year classes to enable detection of regional differences in variance. For yield traits, year variance increased over time as yield of cows increased. Regions were the same as defined by Wiggans and VanRaden (22). These regions, based on geographical proximity and standard deviation for yield by state, may not be optimal for PL.

Value of PL

Economic values of herd-life traits have been reported by several researchers (1, 3, 4, 10, 15, 18). The relative values of yield and herd life derived from these studies (Table 1) indicate that yield is 1 to 8 times as important as herd life. Three studies provided economic values only for herd life adjusted for milk yield. Thus, methods were developed to convert between relative economic values of herd life adjusted for milk yield and unadjusted herd life. Herd life adjusted for milk yield was obtained by regression on within-herd percentile of records for first lactation (17) or last lactation (2, 8) in previous studies. Dekkers et al. (6) used linear, quadratic, and cubic regressions on first lactation deviations from herd-year mean.

The trait APL was defined as PL − .0032(milk yield), where .0032 is the phenotypic regression of PL on first lactation milk yield in kilograms. The regression coefficient was obtained from an earlier multiple-trait REML analysis (19). If milk yield deviation were substituted for milk yield, then rankings of animals would be the same, but the management group solutions would change.

When data are transformed, effects in the model are similarly transformed. Thus, multiple-trait PTA for APL ($\mathbf{PTA_{APL}}$) can be obtained by application of this same transformation to the multiple-trait PTA for PL ($\mathbf{PTA_{PL}}$) and milk ($\mathbf{PTA_{M}}$), if available:

$$PTA_{APL} = PTA_{PL} - .0032\ (PTA_M).$$

This relationship is only approximate for single-trait PTA because some information from correlated traits is lost. Also, the true relationship between yield and PL could be nonlinear. Variance components for APL can be expressed similarly:

$$Var(APL) = Var(PL) + (.0032)^2 Var(milk) - 2(.0032)Cov(APL,milk)$$

Economic values (e) for PL (e_{PL}), APL (e_{APL}), and milk (e_M) have corresponding algebraic relationships. Substitution of the previous identity into the index

$$e_M PTA_M + \dot{e}_{APL} PTA_{APL}$$

gives

$$e_M PTA_M + e_{APL}(PTA_{PL} - .0032\ PTA_M).$$

TABLE 1. Relative economic values of yield and herd life.

Authors	Year	Reference	Relative value[1]	
			Yield	Herd life
Allaire and Gibson	1992	(1)[2]	2.5	1
Congleton and King	1984	(3)	3.9	1
Dekkers	1993	(4)[3]	2.7	1
Harris and Freeman	1993	(10)	8.0	1
Rogers and McDaniel	1989	(15)[4]	.8	1
Van Arendonk	1991	(18)	1.4	1

[1]Value of 1 genetic standard deviation of yield divided by value of 1 genetic standard deviation of herd life.

[2]Original relative value was 3.2:1 for yield:herd life adjusted for milk yield.

[3]Original relative value was 3.4:1 for yield:herd life adjusted for milk yield and was standardized using phenotypic rather than genetic standard deviation.

[4]Original relative value was 1.4:1 for yield:involuntary culling.

Rearrangement of terms gives a second equivalent index:

$$[e_M - .0032(e_{APL})]PTA_M + e_{APL}PTA_{PL}.$$

Economic values of PTA_{PL} and PTA_{APL} are the same ($e_{PL} = e_{APL}$), but the economic value of PTA_M is lower in the second index. Less emphasis and less relative emphasis are placed on PTA_M when the index includes PTA_{PL} because some variation that is due to PTA_M is excluded from PTA_{APL}, which then has less variance. When traits are defined as linear functions of one another, they may be combined with different economic values to produce the same final index (11).

Relative Value of PL

Relative values of traits were determined from their economic values and their genetic variation. Standard deviations of true transmitting abilities were assumed to be 369 kg for milk, 1.92 mo for PL, and 1.71 mo for APL (19). Relative value of milk to APL ($\mathbf{v_{M:APL}}$) is

$$v_{M:APL} = 369(e_M)/1.71(e_{APL}) = 216(e_M/e_{APL}).$$

Relative value of milk to PL ($\mathbf{v_{M:PL}}$) is

$$v_{M:PL} = 369[e_M - .0032(e_{APL})]/1.92(e_{APL}) = 192(e_M/e_{APL}) - .615.$$

Translation from $v_{M:APL}$ to $v_{M:PL}$ can then be achieved by

$$v_{M:PL} = .889(v_{M:APL}) - .615.$$

For example, Allaire and Gibson (1) reported that PTA_M should receive 3.2 times as much emphasis as PTA_{APL}. Equivalently then, PTA_M should receive .889(3.2) – .615 or 2.2 times the emphasis of PTA_{PL} in an index.

Often, only single-trait, rather than multiple-trait, PTA are available. Economic indexes based on single-trait PTA were investigated using a previous data file of VanRaden and Klaaskate (19). Records from 1,984,038 daughters of 2080 sires were included. Any particular economic values could be used to construct an index from PTA_M and PTA_{APL}. Then, multiple regression of that index on PTA_M and PTA_{PL} provides an alternative index based on PTA_{PL}.

TABLE 2. Time required to compute national genetic evaluations for productive life.

Step	Computer used	Time
		(d)
Create and sort data	Mainframe	2
Animal model series	Workstation	4
Merge with yield evaluations	Workstation	2
All[1]	. . .	8

[1]May overlap with other computing tasks.

Literature values for $v_{M:APL}$ were converted to $v_{M:PL}$ using this regression approach rather than the theoretical approach because the current application is to single-trait rather than to multiple-trait PTA. With single-trait evaluation, animals can rank differently, depending on whether adjustment for milk is made before or after BLUP. Correlation of single-trait PTA_{APL} with $PTA_{PL} - .0032(PTA_M)$ was computed in addition to correlations of indexes that combined PTA_M with PTA_{PL} or PTA_{APL}. Correlations were computed for all 2080 sires and separately for the 955 sires with <100 effective daughters to determine whether approximations were poorer with less information. An effective daughter is assumed to have an infinite number of management group mates. Use of PTA_{APL} instead of PTA_{PL} might be preferred because genetic and error covariances with milk are smaller for APL than for PL.

RESULTS AND DISCUSSION

An animal model evaluation of national data on PL was completed; about 4 d were required for the analysis (Table 2). Benefits from evaluation of an additional trait must offset possible delays in release of information for traits that currently are being evaluated. Yield evaluations could be completed more rapidly if evaluations for PL and other traits were processed on another computer or 1 mo earlier than yield evaluations using additional data supplied by dairy records processing centers at that time. Use of new evaluations probably is simplest for the breeding industry if evaluations for all traits are processed and released at the same time.

Mean PL decreased by approximately 2 mo for cows born during the 1980s compared with mean PL during previous decades (Figure 1). Mean genetic merit for PL has increased steadily since 1980 by about .3 mo/yr. With little or no direct selection for PL, this genetic trend reflects previous selection for yield and type, both of which are positively correlated with herd life (17, 19). Positive trend also can result from natural selection, because older cows produce more progeny. Genetic trend for APL was slightly negative and would have been even flatter if APL had been defined with the genetic rather than phenotypic regression coefficient.

As cows have improved, culling standards have been raised. An average cow born in 1967 would be predicted to live about 4 mo less than cows born in 1987 if both had to meet the same culling standard. A downward environmental trend has occurred because cows that once were competitive now would be culled.

Standard deviations within herd for PL data are in Table 3 by region for birth years 1965, 1975, and 1985. Standard deviations were much more consistent across region and year than those observed for milk yield (22); nearly all values round to 13. Procedures to adjust PL for heterogeneous variance were not developed but could still have some benefit because individual herds might differ within regions.

Timeliness and REL of PTA_{PL} were examined for recent animals. Of the 549 bulls in active AI status during January 1993, 507 had at least 10 daughters with PL data. Mean REL for the 507 bulls was 58%. For the remaining 42 bulls, parent averages were available with mean REL of 38%. Thus, mean REL for PL for the 549 active AI bulls was 56%; 6 of the bulls had an REL of 99% for PL. Parent average REL were fairly high because sires and maternal grandsires have REL of 99%, PL records of dams are complete, and maternal brothers may provide additional information for most active AI bulls.

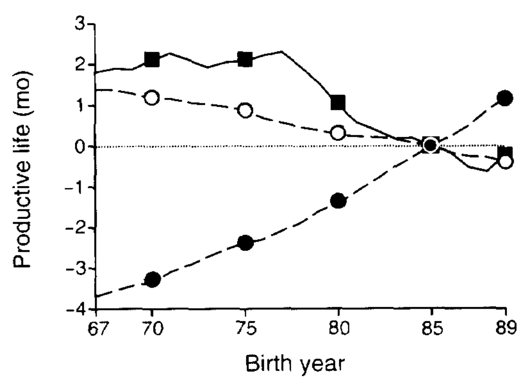

Figure 1. Phenotypic (■) and genetic (●) trends in productive life and productive life adjusted for milk yield (O) relative to a 1985 base of 25 mo by birth year (1967 to 1987).

TABLE 3. Standard deviations of productive life for region and birth year.

	Region		
Birth year	West	Central	East
	(mo)		
1965	12.6	12.9	12.7
1975	12.7	13.4	13.2
1985	13.0	13.7	13.5

The REL for PL, indirect prediction of PL, and protein for active AI bulls are compared in Table 4. Because of the lower heritability, the 3-yr minimum age and the 7-yr wait for complete PL data, REL for PL are 25% lower than for yield and type traits. Boldman et al. (2) and Dekkers et al. (6) used type traits to predict herd life adjusted for milk yield. Boldman et al. (2) reported that REL of predictions for adjusted and unadjusted herd life were similar. Maximum theoretical REL were ≤56%. If REL of type traits averaged 75%, REL of herd life predicted indirectly from type would average only .75(56%) = 42%.

Direct evaluation of herd life captures little or no additional information from young daughters but summarizes ancestor information more completely. For parent averages (Table 4), REL for PL was much higher than REL for indirect prediction of PL from type information and nearly as high as REL for protein yield. A multitrait evaluation could produce higher REL by combining the direct information provided by daughter herd life and the indirect information provided by daughter type and yield traits.

Correlations of PTA_{PL} with PTA for yield and type are in Table 5 along with genetic and phenotypic correlations estimated previously (17, 19). For the population of all bulls born

TABLE 4. Reliabilities (REL) of active AI bulls and their parent averages for protein yield, productive life, and productive life predicted from type.

Evaluation source	REL		
	Protein	Productive life	Productive life from type[1]
	(%)		
Active AI bulls	81	56	42
Parent averages	43	38	24

[1]From Boldman et al. (2).

since 1980 and with ≥10 daughters for each trait, correlations of PTA were close to phenotypic correlations and ranged from .25 to .36. For active AI bulls, correlations were much closer to 0. Lower correlations and lower variances are expected with a selected population. Also, PTA_{PL} was negatively correlated with the percentage of daughters culled during first lactation (13), as expected.

The genetic base was set equal to the genetic merit for PL of the average cow born during 1985, as for yield and type traits. Mean REL was 29% for 3-yr-old cows and 31% for 7-yr-old cows. For active AI bulls, PTA_{PL} ranged from −1.7 to +4.3 mo. The mean PTA_{PL} of +1.2 mo is >0 because of fairly intense selection of these bulls for correlated traits.

Adjustment of PL data for milk yield by phenotypic regression resulted in bull rankings similar to those obtained by adjustment of single-trait PTA_{PL} for PTA_M after genetic evaluation. Correlation of PTA_{APL} with PTA_{PL} − $.0032(PTA_M)$ was .986, which indicates no great advantage to either approach. The correlation remained high, .979, even for bulls with <100 effective daughters. An index that combined PTA_M and PTA_{APL} with relative weights 3.2:1 was correlated by .999 to an index that combined PTA_M and PTA_{PL} with weights 2.5:1. The correlation of these two indexes was also .999 for bulls with <100 effective daughters. Indexes can include PTA_{PL} or PTA_{APL} with equal effectiveness but with somewhat different economic weights.

The relative weighting of 2.5:1 for milk:PL for the second index was chosen to maximize the correlation of the two indexes. Multiple regression of the first index on PTA_M and PTA_{PL} produced the coefficients .00729 and .562, respectively. The ratio of these coefficients was multiplied by the ratio of standard deviations to calculate relative value:

$$v_{M:PL} = (.00729/.562)(369/1.92) = 2.5.$$

The regression approach provides $v_{M:PL}$ somewhat closer to $v_{M:APL}$ than does the theoretical approach, which gave a $v_{M:PL}$ of 2.2 for a $v_{M:APL}$ of 3.2.

Correlation of PTA_{PL} and PTA_{APL} was .831; PTA_{APL} favors bulls with daughters that produce somewhat less milk. Although either of these PTA could contribute to selection indexes, PTA_{PL} was chosen for routine use because it is slightly simpler to compute and to understand. An index of PTA_M and PTA_{PL} with $v_{M:PL}$ = 2.5 was correlated with PTA_M by .964. Economic progress should be 1/.964 or about 4% higher if unbiased PTA_{PL} are included in selection indexes. Benefits would be smaller if indexes already contained type or other nonyield traits that are correlated with PL.

TABLE 5. Correlations of productive life PTA with other traits.

Correlation source	MFP$[1]	PTA Type	Udder composite	Percentage culled
Active AI bull PTA	.08	.11	.24	−.47
Recent bull PTA	.36	.25	.28	−.36
Phenotypic	.35	.23[2]	. . .	. . .
Genetic	.45	.47[2]	. . .	. . .

[1]Economic index for milk, fat, and protein = ($.0605/kg)PTA milk + ($3.26/kg)PTA fat + ($3.15/kg)PTA protein.
[2]From Short and Lawlor (17).

CONCLUSIONS

Evaluations for PL provided sufficient additional information about the economic merit of current animals to justify routine calculation and use. National evaluations for PL were calculated by USDA, and information for bulls was released to the dairy industry for the first time during January 1994. The percentage of daughters culled during first lactation was replaced by mean PL of daughters and PTA_{PL}.

Economic merit can be improved by inclusion of PL or APL in indexes. Literature estimates of $v_{M:PL}$ varied but centered on a relative value of 2.5:1. For USDA-DHIA evaluations for January 1994, PTA for PL and SCS were combined with yield PTA in an economic index called net merit:

$$.7(MFP\$) + [91.9(SMP)(PTA_{PL}) - 229.4(SMP)(PTA_{SCS} - \text{breed SCS})]$$

where MFP$ is the USDA economic index based on PTA for milk, fat, and protein; the factor .7 is an adjustment for feed costs; SMP is the standard milk price; and breed SCS is the mean SCS for first lactations of cows born during 1985 that has been standardized for calving age. The net merit index is equivalent to a relative emphasis of 10:4:–1 for yield:PL: SCS. Beginning in July 1994, the net merit index was used to assign percentile ranking for bulls.

Although REL for PL evaluations were lower than REL for yield or final score evaluation, they were higher than REL of indirect predictions of PL from type traits. Thus, a single-trait evaluation for PL is more valuable in prediction of herd life than are PTA for 14 type traits. However, type traits can add to the accuracy of PL evaluations if both data sources are combined in a multiple-trait evaluation. Selection for PL complements current selection for high yielding cows with desirable type. Genetic gains for PL will allow breeders to increase their culling standards.

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REFERENCES

1 Allaire, F. R., and J. P. Gibson. 1992. Genetic value of herd life adjusted for milk production. J. Dairy Sci. 75:1349.

2 Boldman, K. G., A. E. Freeman, B. L. Harris, and A. L. Kuck. 1992. Prediction of sire transmitting abilities for herd life from transmitting abilities for linear type traits. J. Dairy Sci. 75:552.

3 Congleton, W. R., Jr., and L. W. King. 1984. Profitability of dairy cow herd life. J. Dairy Sci. 67:661.

4 Dekkers, J.C.M. 1993. Theoretical basis for genetic parameters of herd life and effects on response to selection. J. Dairy Sci. 76:1433.

5 Dekkers, J.C.M., and L. K. Jairath. 1994. Requirements and uses of genetic evaluations for conformation and herd life. Proc. 5th World Congr. Genet. Appl. Livest. Prod. XVII:61.

6 Dekkers, J.C.M., L. K. Jairath, and B. H. Lawrence. 1994. Relationships between sire genetic evaluations for conformation and functional herd life of daughters. J. Dairy Sci. 77:844.

7 Ducrocq, V. 1994. Statistical analysis of length of productive life for dairy cows of the Normande breed. J. Dairy Sci. 77:855.

8 Ducrocq, V., R. L. Quaas, E. J. Pollak, and G. Casella. 1988. Length of productive life of dairy cows. 2. Variance component estimation and sire evaluation. J. Dairy Sci. 71:3071.

9 Everett, R. W., J. F. Keown, and E. E. Clapp. 1976. Production and stayability trends in dairy cattle. J. Dairy Sci. 59:1532.

10 Harris, B. L., and A. E. Freeman. 1993. Economic weights for milk yield traits and herd life under various economic conditions and production quotas. J. Dairy Sci. 76:868.

11 Kennedy, B. W., J.H.J. van der Werf, and T.H.E. Meuwissen. 1993. Genetic and statistical properties of residual feed intake. J. Anim. Sci. 71:3239.

12 Madgwick, P. A., and M. E. Goddard. 1989. Genetic and phenotypic parameters of longevity in Australian dairy cattle. J. Dairy Sci. 72:2624.

13 Powell, R. L. 1981. Computation and interpretation of the percentage of first lactation records culled in Sire Summaries. DHI Lett. 57(2):6.

14 Rekik, B., and F. R. Allaire. 1993. Contribution of stayability records to the accuracy of selection for improved production value and herd life. J. Dairy Sci. 76:2299.

15 Rogers, G. W., and B. T. McDaniel. 1989. The usefulness of selection for yield and functional type traits. J. Dairy Sci. 72:187.

16 Rogers, G. W., B. T. McDaniel, M. R. Dentine, and D. A. Funk. 1989. Genetic correlations between survival and linear type traits measured in first lactation. J. Dairy Sci. 72:523.

17 Short, T. H., and T. J. Lawlor. 1992. Genetic parameters of conformation traits, milk yield, and herd life in Holsteins. J. Dairy Sci. 75:1987.

18 Van Arendonk, J.A.M. 1991. Use of profit equations

to determine relative economic value of dairy cattle herd life and production from field data. J. Dairy Sci. 74:1101.

19 VanRaden, P. M., and E.J.H. Klaaskate. 1993. Genetic evaluation of length of productive life including predicted longevity of live cows. J. Dairy Sci. 76: 2758.

20 VanRaden, P. M., and G. R. Wiggans. 1991. Derivation, calculation, and use of national animal model information. J. Dairy Sci. 74:2737.

21 VanRaden, P. M., G. R. Wiggans, and C. A. Ernst. 1991. Expansion of projected lactation yield to stabilize genetic variance. J. Dairy Sci. 74:4344.

22 Wiggans, G. R., and P. M. VanRaden. 1991. Method and effect of adjustment for heterogeneous variance. J. Dairy Sci. 74:4350.